

Complex Numbers

Imaginary Number: $i = \sqrt{-1}$ and $i^2 = -1$

Complex Numbers: The set of all numbers of the form $a + bi$ where a and b are real numbers.

a is called the **real part** and b is called the **imaginary part**. If $b \neq 0$, then $a + bi$ is an **imaginary number**. The form $a + bi$ is called the **standard form** of a complex number.

Examples: Determine whether each complex number is real or imaginary and write it in standard form.

a) $4i$

b) $3 - 6i$

c) 5

d) $\frac{i - 3\pi}{4}$

Addition, Subtraction, and Multiplication of Complex Numbers.

$$(a + bi) + (c + di) = (a + c) + (b + d)i$$

$$(a + bi) - (c + di) = (a - c) + (b - d)i$$

$$(a + bi)(c + di) = (ac - bd) + (bc + ad)i \text{ or use FOIL.}$$

Examples: Perform the indicated operations.

a) $(6 + 2i) + (4 - 3i) =$

b) $(7 - 4i) - (-2 + 8i) =$

c) $(6 + 5i)(8 + 3i) =$

d) $(1 - i)(4 + i) =$

Powers of i :

Since $i^2 = -1$, $i^3 = i^2 \cdot i = -1 \cdot i = -i$, and $i^4 = i^2 \cdot i^2 = -1 \cdot -1 = 1$

The first eight powers are listed here:

$$i^1 = i \qquad i^5 = i$$

$$i^2 = -1 \qquad i^6 = -1$$

$$i^3 = -i \qquad i^7 = -i$$

$$i^4 = 1 \qquad i^8 = 1$$

The powers of i continue in this pattern.

Examples: Simplify the power of i .

a) $i^{35} =$

b) $i^{29} =$

c) $i^{98} =$

d) $i^{48} =$

Theorem: If a and b are real numbers, then the product of $a + bi$ and its conjugate $a - bi$ is the real number $a^2 + b^2$. $(a + bi)(a - bi) = (a^2 + b^2)$

Examples: Find the product of the complex number and its conjugate.

a) $3 - 7i$

b) $2 + 9i$

c) i

Examples: Write each quotient in the form $a + bi$.

a) $\frac{6 - 2i}{3} =$

b) $\frac{2}{8 + 9i} =$

c) $\frac{4 - 5i}{3 + 2i} =$

Roots of Negative Numbers

For any positive real number b , $\sqrt{-b} = i\sqrt{b}$.

Examples: Write each expression in the form $a + bi$, where a and b are real numbers.

a) $\sqrt{-5} + \sqrt{-8} =$

b) $\sqrt{-20}(\sqrt{-6} - \sqrt{-4}) =$

c) $\frac{-2 + \sqrt{-48}}{2} =$

Example: Does the complex number $x = 1 + 3i\sqrt{2}$ satisfies the equation $x^2 - 2x + 4 = 0$?